NCF-003-016204

Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May – 2017

Course - 2004 : Mathematics

(Methods in Partial Differential Equation)

[Old Course]

Faculty Code : 003

Subject Code : 016204

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) There are **five** questions.

(ii) **All** questions are **compulsory**.

(iii) **Each** question carries **14** marks.

1 Do as directed : (Each question carries **two** marks) **14**

(a) Find the complementary function of the equation $\left(D^3+3D^2D'-4D'^3\right)z=0$.

(b) Eliminate the arbitrary constants a and b from $2z=(ax+y)^2+b$.

(c) Define Pfaffian differential equation with an example and write down the condition for the pfaffian differential to be integrable.

(d) Find the complete integral of $yp-xq=0$.

(e) Find the P.d.e. of $z=f\left(\frac{xy}{z}\right)$ where f is an arbitrary function.

(f) Find curl $\left(7y^2z^2,\ 5x^2z^2,\ 9x^2y^2\right)$.

(g) Classify the equation $4y^2r+x^2t=0$.

2 Answer any two of the following : **2×7=14**

(a) Find the general solution to

$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. Show that u contains in the combination of $x+y+z$ and $x^2+y^2+z^2$.

(b) Find the orthogonal trajectories on the conicoid $(x+y)z=1$ of the conics in which it is cut by the system of planes $x-y+2=k$ where k is the parameter.

(c) Prove that a pfaffian differential equation

$$\left(xy^2-zx\right)dy+yzdx=-\left(xz^2-yx\right)dz$$

is integrable. Also find the complete primitive.

3 All are compulsory : **14**

(a) (i) Determine the envelope of one parameter of system of the surfaces $x^2+y^2+(z-a)^2=1$. **4**

(ii) Determine the envelope of two parameter of system of the surfaces $(x-a)^2+z^2+(y-a)^2=1$.

(b) Find the integral curves of the equation **5**

$$\frac{dx}{(x+z)}=\frac{dy}{y}=\frac{dz}{\left(z+y^2\right)}.$$

(c) Prove that **5**

$$F(D,D')\left[e^{ax+by}\cdot h(x,y)\right]=e^{ax+by}F(D+a,D'+b)\left[h(x,y)\right].$$

OR

3 All are compulsory : **14**

(a) Solve the equation : **4**

(i) $z(z+y)dx+z(z+x)dy-2xydz=0$.

(ii) $(1+x)yzdx+(1+y)xzdy+(1+z)xydz=0$.

(b) Prove necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $f(u,v)=0$ not involving x and y explicility is that $\frac{\partial(u,v)}{\partial(x,y)}=0$. **5**

(c) Using Natani's method **5**

$x(y^2-1)dx+y(x^2-z^2)dy-z(y^2-1)dz=0$.

4 Answer any two of the following : **2×7=14**

(a) Solve $p=(z+qy)^2$.

(b) Classify the equation and convert it into canonical form $x^2r+4t=xy$. $(x\neq 0)$.

(c) If $(\beta D'+\gamma)^2$ with $\beta\neq 0$ is a factor of $F(D,D')$, then a solution of the equation $F(D,D')$ is,

$$z=e^{\frac{-\gamma}{\beta}y}\left(\varnothing_1(\beta x)+y\varnothing_2(\beta x)\right)$$

Where $\varnothing_i=\varnothing_i(\varepsilon)$ is an arbitrary function of a single variable $(i=1,2)$.

5 Answer any two of the following : **2×7=14**

(a) Find the G.S. of $(2D+3D')(D-D')z=\frac{(x^2+xy)}{2}$.

(b) (i) Find the solution of $(x^2D^2-y^2D'^2+xD-yD')z=0$.

(ii) Using Jacobi's method solve $xp^2+yq^2=z$.

(c) Find the equation of integral surface of the differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ passes through $y=0$ and $x=1$.